Annihilation processes in random velocity field

T. Lučivjanský[1]

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[1]PhD student at Pavol-Jozef Šafárik University in Košice, Slovakia
visiting at the University of Vienna thanks to the Ernst-Mach scholarship provided by OeAD

Outline

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- reaction-diffusion problems with particles localized in space
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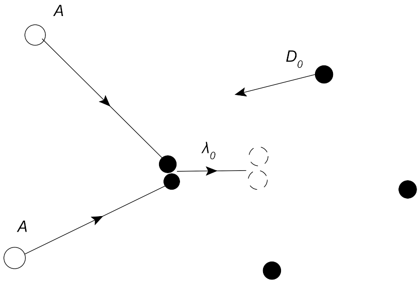

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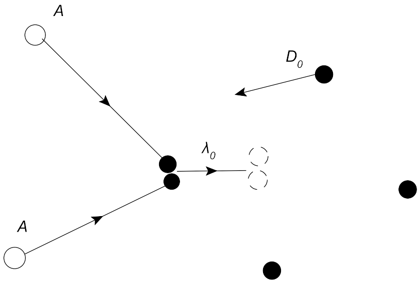

- particles can be interpreted as molecules, biological entities, mutually annihilating random walk etc.

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  let $d \leq 2$, in this case the diffusion is recurrent
  displacement $r(t) \sim (Dt)^{1/2}$ and particles “sweeps“ the volume
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- **Corollary** spatial dimension $d = 2$ plays a special role for this process

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- the correct eq. should be

$$\partial_t n = D_0 \nabla^2 n - \lambda_0 n^{(2)},$$

where $n^{(2)}$ is probability of finding 2 particles at one site

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- by describing properties of **v** one can model thermal fluctuations, external stirring, turbulent state etc.

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- **1)** short-range correlations

$$\langle v_i v_j \rangle \propto \delta_{ij} \delta(t - t') \delta(\mathbf{x} - \mathbf{x}')$$

M. J. E. Richardson, J. Cardy J. Phys. A 32 4035-4045 (1999)