Influence Of The Compressibility On The Anomalous Kinetics Of The Annihilation Process

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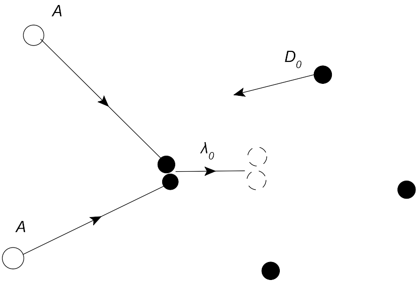
particles are diffusing with diffusion constant D_0 and reacting after mutual contact



irreversible reaction $A + A \xrightarrow{\lambda_0} \varnothing$

$n(t) = ?$

reaction limited...$\tau_{dif} \ll \tau_{react}$

corresponds to high D_0 or small λ_0

no spatial fluctuations $\Rightarrow n = n(t)$

Rate equation $\frac{dn(t)}{dt} = -\lambda_0 n^2(t) \rightarrow n(t) \propto t^{-1}$

diffusion limited...$\tau_{dif} \gg \tau_{react}$

corresponds to small D_0 or high λ_0

displacement $r(t) \sim (Dt)^{1/2}$

for $d \leq 2$ (diffusion is recurrent) $V(t) \sim r(t)^d$
$n(t) \sim (Dt)^{-d/2} = (Dt)^{-(1+\delta)}$, where $d = 2 + 2\delta$

for $d > 2$ $\quad V(t) \sim t \Rightarrow n(t) \sim t^{-1}$

mean-field approach

$$\partial_t n = D_0 \nabla^2 n - \lambda_0 n^2$$

Our goal is to include advection

$$\partial_t \to \nabla_t = \partial_t + (\mathbf{v}.\nabla)$$
$$\partial_t n + (\mathbf{v}.\nabla) n = D_0 \nabla^2 n - \lambda_0 n^2$$

v can model thermal fluctuations, external stirring, turbulent state etc.

Consider $\mathbf{v}(t, \mathbf{x})$ - random gaussian variable

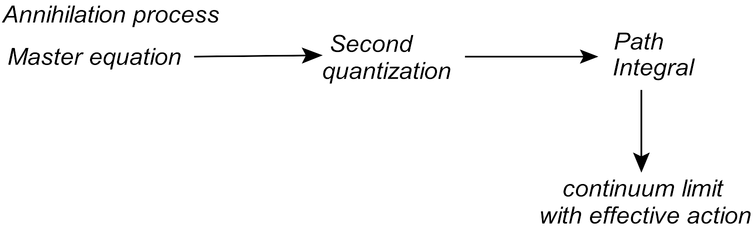
$$\langle \mathbf{v} \rangle = 0$$
$$\langle v_i v_j \rangle(\omega, \mathbf{k}) = \{P^k_{ij} + \alpha Q^k_{ij}\} D_v(\omega, \mathbf{k}),$$
$$P^k_{ij} = \delta_{ij} - k_i k_j / k^2 \quad \text{transversal}$$
$$Q^k_{ij} = k_i k_i / k^2 \quad \text{longitudinal}$$

α - degree of compresibility

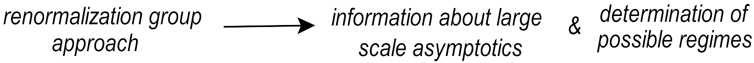
$$D_v(\omega, \mathbf{k}) \;\propto\; \frac{k^{4-d-2\epsilon-\eta}}{\omega^2 + [u_0 D_0 k^{2-\eta}]^2}$$

energy spectrum $E(k) \propto k^{1-2\epsilon}$
characteristic frequency of the mode $\mathbf{k}$ is $\omega_k \propto k^{2-\eta}$
rapid-change model $D_v(\omega, \mathbf{k}) = D_v(\mathbf{k})$
$\langle \mathbf{v}(t)\mathbf{v}(t') \rangle \propto \delta(t - t')$
frozen-velocity field $D_v(\omega, \mathbf{k}) \propto \delta(\omega)$
$\langle \mathbf{v}(t)\mathbf{v}(t') \rangle \not\propto (t - t')$

Overview of the approach



At the very end



Main result of master eq. approach

action S for $A + A \to \varnothing$

$$S = \psi^\dagger[-\partial_t\psi - (\mathbf{v}.\nabla)\psi + D_0\nabla^2\psi] - \lambda_0 D_0[2\psi^\dagger + (\psi^\dagger)^2]\psi^2 + n_0\psi^\dagger(\mathbf{x},0) - \frac{1}{2}\mathbf{v}D_v^{-1}\mathbf{v} \quad (1)$$

all integrations are omitted, e.g.

$\psi^\dagger\partial_t\psi = \int dt \int d\mathbf{x}\psi^\dagger(t,\mathbf{x})\partial_t\psi(t,\mathbf{x})$

$$\text{time average } A(t) = \int \mathcal{D}\psi^\dagger\mathcal{D}\psi\mathcal{D}\mathbf{v}\, A\, \mathrm{e}^S \quad (2)$$

Results

triple expansion in (δ, ϵ, η), where

$\delta = (d-2)/2$

ϵ - deviation from the Kolmogorov scaling

η - from the parabolic dispersion law for frequency

prediction of Kolmogorov 41 theory obtained for the choice $\epsilon = \eta = 4/3$

density decay rate $n(t) \propto t^{-\beta}$

rate equation prediction $\beta = 1$ (mean-field like)

Rapid-change model

$\langle v(t)v(t')\rangle \propto \delta(t-t')$

FP	$g^{*\prime}$	λ^*	β	Stable for
1A	0	0	1	$\eta > 2\epsilon, \eta > 0, \delta > 0$
1B	0	$-4\pi\delta$	$1+\delta$	$\eta > 2\epsilon, \eta > 0, \delta < 0$
2A	$\frac{8\pi(2\epsilon-\eta)}{1+\alpha}$	0	$1+\frac{\alpha}{1+\alpha}\frac{2\epsilon-\eta}{2-2\epsilon+\eta}$	$2\epsilon > \eta > \epsilon, 2\delta > \frac{\eta-2\epsilon}{1+\alpha}$
2B	$\frac{8\pi(2\epsilon-\eta)}{1+\alpha}$	$-4\pi\delta-\frac{2\pi(2\epsilon-\eta)}{1+\alpha}$	$1+\frac{2\delta+2\epsilon-\eta}{2-2\epsilon+\eta}$	$2\epsilon > \eta > \epsilon, \frac{\eta-2\epsilon}{1+\alpha} > 2\delta$

Frozen-velocity field

$\langle v(t)v(t')\rangle \not\approx (t-t')$

FP	$g^{*\prime\prime}$	λ^*	β	Stable for
3A	0	0	1	$\epsilon < 0, \eta < 0, \delta > 0$
3B	0	$-4\pi\delta$	$1+\delta$	$\epsilon < 0, \eta < 0, \delta < 0$
4A	$8\pi\epsilon$	0	$1+\frac{\alpha\epsilon}{2-\epsilon(1-\alpha)}$	$\epsilon > 0, \epsilon(1-\alpha) > \eta, 2\delta+\epsilon(1-2\alpha) > 0$
4B	$8\pi\epsilon$	$-4\pi\delta-2\pi\epsilon(1-2\alpha)$	$1+\frac{2\delta+\epsilon(1-\alpha)}{2-\epsilon(1+\alpha)}$	$\epsilon > 0, \epsilon(1-\alpha) > \eta, 2\delta+\epsilon(1-2\alpha) < 0$

Non-trivial fixed points

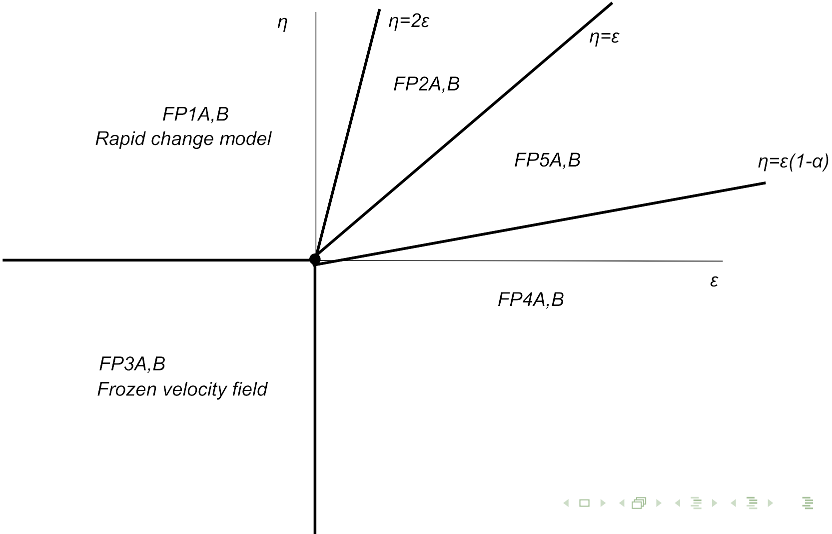
for both of them

$$\frac{g^*}{8\pi u^*(1+u^*)} = \frac{2\epsilon - \eta}{1+\alpha},$$

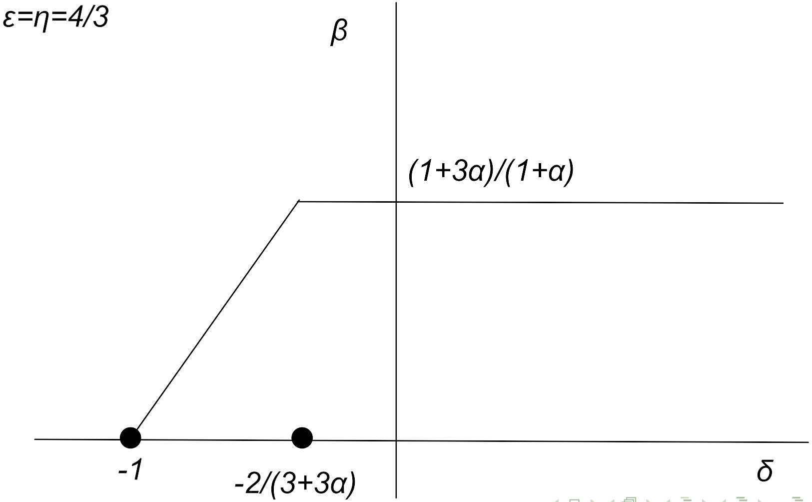
$$u^* = -1 + \frac{\alpha(\eta - 2\epsilon)}{(1+\alpha)(\eta - \epsilon)}$$

FP	λ^*	β	Stable for
5A	0	$1 + \frac{\alpha}{1+\alpha}\frac{2\epsilon-\eta}{2-\eta}$	$\epsilon > \eta > \epsilon(1-\alpha), \delta > \frac{2\epsilon\alpha-\eta(1+2\alpha)}{2(1+\alpha)}$
5B	$-4\pi\delta + \frac{2\pi}{1+\alpha}[\alpha\epsilon - \eta(1+\alpha)]$	$1 + \frac{2\delta+\eta}{2-\eta}$	$\epsilon > \eta > \epsilon(1-\alpha), \delta < \frac{2\epsilon\alpha-\eta(1+2\alpha)}{2(1+\alpha)}$

Phase diagram for $\delta = 0$



Turbulent state



Summary

Conclusions

- field-theoretic model for annihilation process in the presence of compressible velocity field was constructed
- compresibility tends to increase progress of annihilation process
- density fluctuations acts against compresibility
- fixed points and thus large-scale behaviour were estimated to the leading order

Thank you for your attention